

CAMBRIDGE INTERNATIONAL EXAMINATIONS
GCE Ordinary Level

MARK SCHEME for the May/June 2013 series

4037 ADDITIONAL MATHEMATICS

4037/21

Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Accuracy mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

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| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| SOS | See Other Solution (the candidate makes a better attempt at the same question) |

Penalties

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| MR –1 | A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. |
| OW –1, 2 | This is deducted from A or B marks when essential working is omitted. |
| PA –1 | This is deducted from A or B marks in the case of premature approximation. |
| S –1 | Occasionally used for persistent slackness – usually discussed at a meeting. |
| EX –1 | Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting. |

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| <p>1</p> | $\frac{2 + 2 \sin^2 \theta}{\cos^2 \theta}$ $\frac{2}{\cos^2 \theta} = 2 \sec^2 \theta$ $\frac{\sin^2 \theta}{\cos^2 \theta} = 2 \tan^2 \theta$ <p>$2 \sec^2 \theta = 2 + 2 \tan^2 \theta$ and completion</p> <p>Or</p> $(\sec \theta + \tan \theta)^2 + (\sec \theta - \tan \theta)^2$ $2 \sec^2 \theta + 2 \tan^2 \theta$ $2(1 + \tan^2 \theta) + 2 \tan^2 \theta$ and completion <p>Or</p> $\frac{2 + 2 \sin^2 \theta}{\cos^2 \theta}$ $\frac{2(\sin^2 \theta + \cos^2 \theta) + 2 \sin^2 \theta}{\cos^2 \theta}$ $\frac{4 \sin^2 \theta}{\cos^2 \theta} = 4 \tan^2 \theta$ $\frac{2 \cos^2 \theta}{\cos^2 \theta} = 2$ and completion | <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[B1, B1]</p> <p>B1</p> <p>B1]</p> <p>[B1</p> <p>B1</p> <p>B1</p> <p>B1]</p> | <p>For all methods look for:</p> <ul style="list-style-type: none"> – correct simplified expression – correct use of Pythagoras – use of $\tan = \frac{\sin}{\cos}$ – use of $\frac{1}{\cos} = \sec$ <p>Award first 3 then last B1 for final expression from fully correct method.</p> <p>Inconsistent no angle used then –1 (can recover).</p> <p>If start from RHS award similarly.</p> |
| <p>2</p> | <p>(i) 3.2</p> <p>(ii) 15</p> <p>(iii) uses area to find distance</p> <p>two of 40, 240 and 32</p> <p>312</p> | <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> | <p>If split 2 or 3 correct formulae and must be attempting total area</p> <p>or A2 for 312 from trapezium</p> |

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| 3 | $\frac{dy}{dx} = k \sin x \cos x$ $k = -8$ <p>Attempt to find x when $y = 8$</p> $x = \frac{\pi}{4} \text{ (0.785)}$ <p>Uses $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$</p> $-0.8 \text{ (not rounded)}$ | <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> | <p>Must get to $x =$ numerical value</p> <p>$45^\circ = \mathbf{A0}$ (but can still gain next 2 marks)</p> <p>Must use numerical value for x and 0.2 for $\frac{dx}{dt}$</p> <p>(condone poor notation if correct terms multiplied)</p> |
| 4 | <p>(i) Idea of modulus correct</p> <p>$\frac{1}{2}$ indicated on x-axis</p> <p>2 indicated on y-axis</p> <p>(ii) $\frac{2}{3}$ (0.667)</p> <p>Solve $4x - 2 = -x$ or $(4x - 2)^2 = x^2$</p> <p>$\frac{2}{5}$</p> | <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> | <p>Two straight lines above and touching x-axis</p> <p>Must be a sketch</p> <p>Must be a sketch</p> <p>0.67 is B0</p> <p>As far as $x =$ numerical value</p> <p>SC: If drawn then B1, B2 for exact answers only</p> |
| 5 | <p>(i) $(QR = PS) = \frac{96 - 3x}{2}$</p> <p>Area = $\left(\frac{96 - 3x}{2}\right) \times x$</p> <p>(ii) $\frac{dA}{dx} = \frac{96 - 6x}{2}$ or $48 - 3x$ o.e.</p> <p>Solving $\frac{dA}{dx} = \frac{96 - 6x}{2} = 0$</p> <p>$x = 16$</p> <p>$A = 384$ and state maximum</p> | <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> | <p>Can be implied by next statement</p> <p>AG</p> <p>As far as $x =$ numerical value</p> |

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| 6 | <p>Applies quotient rule correctly</p> $\frac{(x-2)2x - (x^2+8)}{(x-2)^2}$ <p>$y = 12$</p> <p>Uses $m_1m_2 = -1$</p> <p>(Gradient normal = $\frac{1}{2}$)</p> <p>Uses equation of line for normal</p> $y - 12 = \frac{1}{2}(x - 4) \quad \text{or} \quad y = \frac{1}{2}x + 10$ | <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> | <p>or product rule</p> $2x(x-2)^{-1} - (x^2+8)(x-2)^{-2}$ <p>If uses $y = mx + c$ must find c for M1</p> |
| 7 | <p>(i) $64 + 192x + 240x^2 + 160x^3$ mark final answer</p> <p>(ii) Multiply out $(1 + 3x)(1 - x)$</p> <p>$1 + 2x - 3x^2$ o.e.</p> <p>$(1) \times (160) + (2) \times (240) + (-3) \times (192)$ o.e.</p> <p>64</p> <p>Or</p> <p>Multiply out $(1 - x)(64 + 192x + 240x^2 + 160x^3)$</p> <p>...$48x^2 - 80x^3$...o.e.</p> <p>Multiply by $1 + 3x$</p> <p>64</p> <p>Or</p> <p>$(1 + 3x)(64 + 192x + 240x^2 + 160x^3)$</p> <p>...$816x^2 + 880x^3$...o.e.</p> <p>Multiply by $1 - x$</p> <p>64</p> | <p>B3, 2, 1, 0</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[M1]</p> <p>A1</p> <p>M1</p> <p>A1]</p> <p>[M1]</p> <p>A1</p> <p>M1</p> <p>A1]</p> | <p>3 terms correct earn B2; 2 terms correct earn B1 Can be earned in (ii); SC2 correct but unsimplified</p> <p>3 terms</p> <p>May be other variations: for first M1 find x^2 term or x^3 term</p> <p>for second M1 must produce all relevant terms</p> |

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| 8 | <p>Eliminates y (or x) and full attempt at expansion</p> <p>$4x^2 - 8x - 96 = 0$ or $y^2 + 12y - 64 = 0$</p> <p>Factorise 3 term relevant quadratic</p> <p>$x = -4$ and 6 or $y = -16$ and 4</p> <p>$y = -16$ and 4 or $x = -4$ and 6</p> <p>Uses Pythagoras for relevant points</p> <p>22.4 or $\sqrt{500}$ or $10\sqrt{5}$</p> | <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1√</p> <p>M1</p> <p>A1</p> | <p>Or use correct formula</p> <p>cao</p> |
| 9 | <p>(i) Attempt to solve 3 term quadratic</p> <p>-3 and 8</p> <p>$-3 < x < 8$</p> <p>(ii) $4 < x (< 12)$</p> <p>$S \cup T = -3 < x < 12$</p> <p>(iii) $S \cap T = 4 < x < 8$ or</p> <p>$S' = -5 < x \leq -3, 8 \leq x < 12$ and</p> <p>$T' = -5 < x \leq 4$</p> <p>$-5 < x \leq 4$</p> <p>$8 \leq x < 12$</p> | <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1√</p> <p>B1√</p> | <p>Condone $-3 < x$ AND $x < 8$</p> <p>Penalise confusion over $<$ and \leq (or $>$ and \geq) once only</p> <p><i>their 4</i></p> <p><i>their 8 (Ignore AND/OR etc.)</i></p> |

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| 10 | (i) | $\frac{\sin \alpha}{50} = \frac{\sin 58}{240}$ $\alpha = 10.2$ Bearing (0)21.8 or (0)22 | M1 A1 | Use of sin rule/cosine rule/resolving with 50, 240 and 58/32/122/148. Must be correct for A1 |
| | (ii) | $V^2 = 240^2 + 50^2 - 2 \times 240 \times 50 \times \cos(122 - \alpha)$ $V = 263 \text{ awt}$ $T = \frac{500}{V}$ 114 or 1 hour 54 mins Or $T = \frac{500 \cos 32}{240 \cos 21.8}$ 500 cos 32 240 cos 21.8 114 or 1 hour 54 mins | A1 A1√ M1 A1 M1 A1 [M1 B1 B1 A1] | √ for 32 – α Correct use of sin rule/cosine rule/resolving Can be in (i) Only allow if V calculated from non right-angled triangle Do not allow incorrect units Alternative for part (ii) only Also can find distance for 240 (457) then 457/240 |
| 11 | (i) | 1 | B1 | Not a range for k, but condone x = 1 and x ≥ 1 |
| | (ii) | f ≥ -5 | B1 | Not x, but condone y |
| | (iii) | Method of inverse $1 + \sqrt{x+5}$ | M1 A1 | Do not reward poor algebra but allow slips Must be f ⁻¹ = ...or y = ... |
| | (iv) | f: Positive quadratic curve correct range and domain f ⁻¹ : Reflection of f in y = x | B1 B1√ | Must cross x-axis √their f(x) sketch Condone slight inaccuracies unless clear contradiction. |
| | (v) | Arrange f(x) = x or f ⁻¹ (x) = x to 3 term quadratic = 0 4 only www | M1 A1 | Allow x = 4 with no working. Condone (4, 4). Do not allow final A mark if -1 also given in answer |

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| 12 | (i) | $f(3) = (27 + 9 + 3a + b) = 0$ or $3a + b = -36$ $f(-1) = (-1 + 1 - a + b) = 20$ or $-a + b = 20$ Solve equations $a = -14, b = 6$ | M1 Equate $f(3)$ to 0 M1 Equate $f(-1)$ to 20 M1 A1 If uses $b = 6$ then M0, A0 Need both values for A1 |
| | (ii) | Find quadratic factor $x^2 - 4x - 2$ Use quadratic formula or completing square on relevant 3 term quadratic $\frac{-4 \pm \sqrt{16 + 8}}{2}$ or better $-2 \pm \sqrt{6}$ isw | M1 If division, must be complete with first 2 terms correct If writes down, must be $(x^2 + kx - 2)$ A1 M1 If completing square, must reach $\left(x + \frac{k}{2}\right)^2 = 2 \pm \left(\frac{k}{2}\right)^2$ A1√ A1 cao |